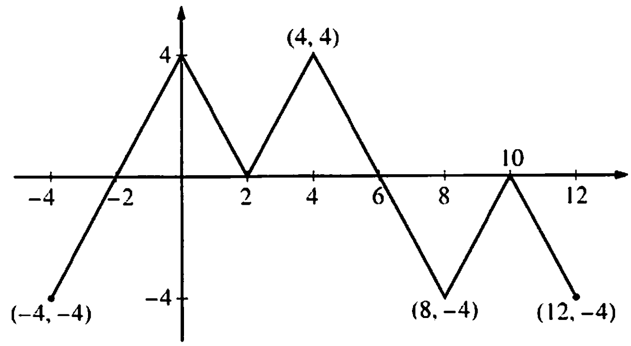
Problem 1 [2016 AB FRQ #3a]



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

$g'(x) = f(x)$

neither since the graph does not change signs

Problem 2 [2015 AB FRQ #1c]

1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

$A(t) = 30 + \int_0^t R(x) - D(x)\,dt$

$A'(t) = R(t) - D(t) = 0$

$t = 3.2717$ hours

$A''(3.2717) = 1.2727 > 0$

so locates min by 2nd deriv test

Problem 3 [2015 AB FRQ #4c]

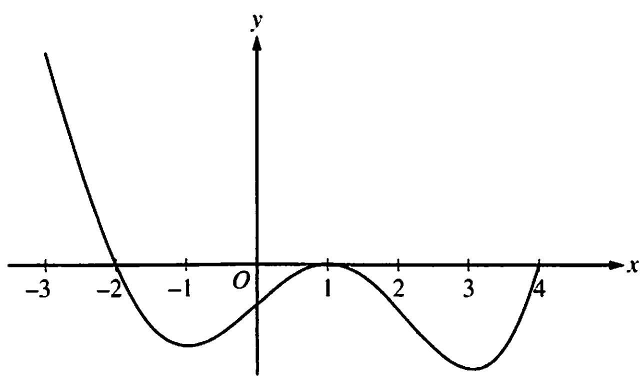
4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

$\left.\frac{dy}{dx}\right|_{(2,3)} = 2(2) - 3 = 1$

neither

Problem 4 [2015 AB FRQ #5a]



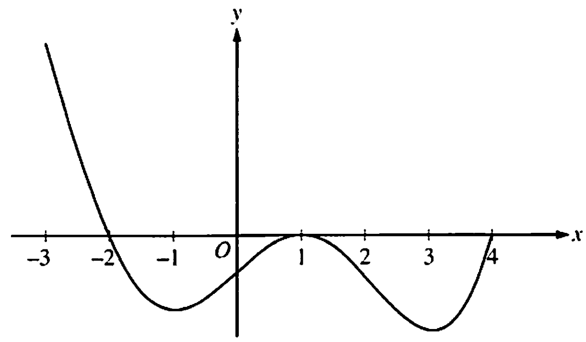
Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

f has relative maximum @ $x = -2$
because f' changes signs

Problem 5 [2015 AB FRQ #5c]



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

$x = -1, 1, 3$ because f changes
from concave up to concavedown or
vice versa

Problem 6 [2014 AB FRQ #5a]

x	−2	$-2 < x < -1$	−1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	−5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	−1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	−2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

   (a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

$x = 1$ because sign changes from
negative to positive

Problem 7 [2013 AB FRQ #1d]

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

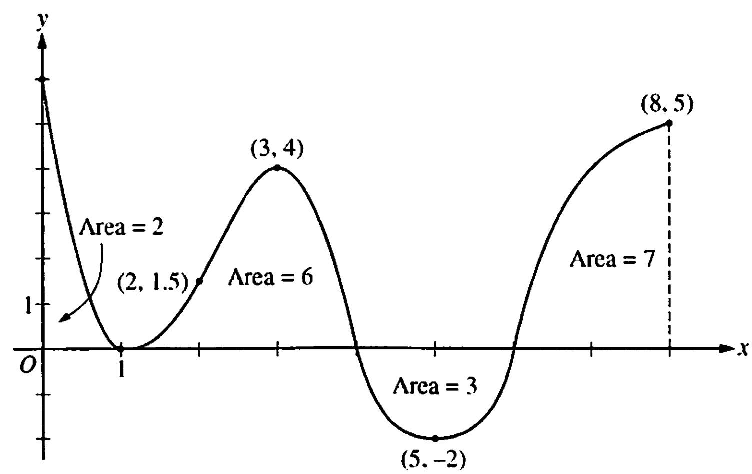
$A(t) = 500 + \int_0^t G(x) - 100\,dx$

$A'(t) = G(t) - 100 = 0$

$t = 4.923$ only crit pt in $[0, 8]$

max of 635.376 tons of unprocessed gravel at $t = 4.923$ hours by first deriv. test

Problem 8 [2013 AB FRQ #4a]



Graph of f'

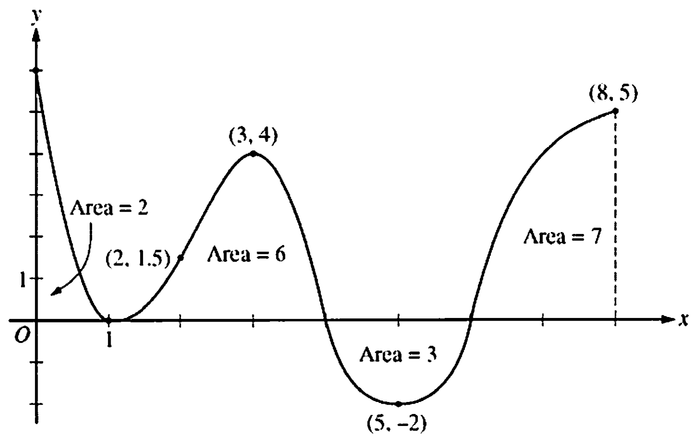
4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

$x = 6$ because f changes from dec to inc.

Problem 9 [2013 AB FRQ #4b]

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.



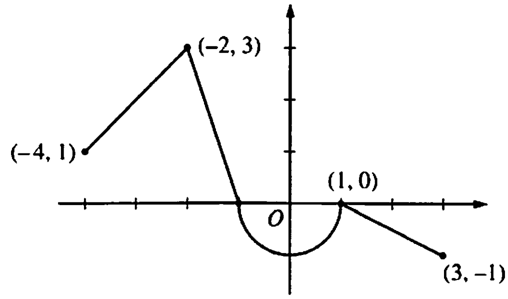
Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

x	F(x)
0	-8
1	-6
4	0
6	-3
8	4 given

min value occurs at $x = 0$

Problem 10 [2012 AB FRQ #3c]



Graph of f

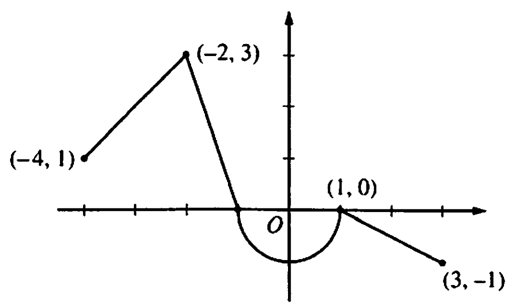
3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

$x=-1$ rel max b/c $g'=f$ crosses x-axis

$x=1$ neither b/c $g'=f$ does not change signs

Problem 11 [2012 AB FRQ #3d]



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

$x = -2, 0, 1$ because $g' = f$ changes direction